Midterm Exam I: MAT 310

Instructions: Complete all problems below. You may not use calculators or other electronic devices, including cell phones. Show all of your work. **Be sure to write your name and student ID on each page that you hand in.**

1.(15pts) If U and W are subspaces of a vector space V, show that $U \cup W$ need not be a subspace. However, if $U \cup W$ is a subspace, show that either $U \subset W$ or $W \subset U$.

2.(14pts) Solve the following system of linear equations.

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 5 \\ x_1 + 4x_2 - 3x_3 - 3x_4 &= 6 \\ 2x_1 + 3x_2 - x_3 + 4x_4 &= 8 \end{aligned}$$

3.(13pts) Determine whether or not $\{(1,1,0),(2,0,-1),(-3,1,1)\}$ is a basis for $\mathbb{R}^3$. Justify your answer.

4.(15pts) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ denote a linear transformation such that $T((1,0,0,0)) = (3,-1,0)$, $T((1,1,1,1)) = (-2,1,3)$, and $T((0,0,1,1)) = (0,1,1)$. Compute the dimension of the null space $dim(N(T))$.

5.(15pts) Let $\alpha = \{1, x, x^2\}$ and $\beta = \{x^2+x, x-1, x^2+x+1\}$ be two different bases for the vector space of polynomials $P_2(\mathbb{R})$. Compute the change of coordinates matrix Q which changes α-coordinates to β-coordinates. Next, compute the coordinates $[1+x]_\beta$.

6.(15pts) Let $T : V \to W$ and $L : W \to Z$ be linear transformations between vector spaces V, W, and Z. Prove that if LT is onto then L is onto. Must T also be onto?

7.(13pts) Let C^∞ denote the vector space (over the complex numbers) of infinitely differentiable complex valued functions defined on the real line. Find the general solution to the homogeneous differential equation $p(D)y = 0$ where

$$p(x) = (x^2+1)^2(x^2+4x+3).$$